

Up to eleven: radiation from particles with arbitrary energy falling into higher-dimensional black holes

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We consider point particles with arbitrary energy per unit mass E that fall radially into a higher-dimensional, nonrotating, asymptotically flat black hole. We compute the energy and linear momentum radiated in this process as functions of E and of the spacetime dimensionality $D = n + 2$ for $n = 2, \dots, 9$ (in some cases we go up to 11). We find that the total energy radiated increases with n for particles falling from rest ($E = 1$). For fixed particle energies $1 < E \leq 2$ we show explicitly that the radiation has a local minimum at some critical value of n , and then it increases with n . We conjecture that such a minimum exists also for higher particle energies. The present point-particle calculation breaks down when $n = 11$, because then the radiated energy becomes larger than the particle mass. Quite interestingly, for $n = 11$ the radiated energy predicted by our calculation would also violate Hawking's area bound. This hints at a qualitative change in gravitational radiation emission for $n \gtrsim 11$. Our results are in very good agreement with numerical simulations of low-energy, unequal-mass black hole collisions in $D = 5$ (that will be reported elsewhere) and they are a useful benchmark for future nonlinear evolutions of the higher-dimensional Einstein equations.

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I. INTRODUCTION

The dynamics of black holes (BHs) in generic spacetimes has attracted considerable attention in recent years. In astrophysics, BHs are important as sources of gravitational and electromagnetic waves. The inspiral and merger of BH binaries is a primary target for Earth-based and space-based gravitational-wave detectors [1]. In gas-rich environments, BH mergers may be associated with detectable electromagnetic precursors or afterglows [2, 3] and even drive the production of jets [4]. In high-energy physics, the gauge/gravity duality [5, 6] has created a powerful framework for the study of strongly coupled gauge theories, with applications in connection with the experimental program on heavy ion collisions at RHIC [7] and LHC [8, 9], among many others. The fact that BHs on the gravitational side of this correspondence are dual to thermal states of the gauge theory has sparked a renewed interest in BH physics. Furthermore, some proposals to solve the hierarchy problem postulate the existence of extra dimensions accessible only to gravity [8, 9]. In these scenarios, BH production from the collision of particles at energy scales above TeV is an almost inescapable consequence.

Gravitational wave detection and high-energy applications require an accurate knowledge of BH dynamics and gravitational radiation emission. This triggered research on the numerical evolution of the full nonlinear Einstein equations in four [10–18] and higher dimensions [19–23]. The validation of numerical codes requires semianalytical tools, such as post-Newtonian theory, BH perturbation theory and zero-frequency expansions to model BH collisions. Such tools have been available for decades in the case of four-dimensional, asymptotically flat spacetimes

(see e.g. [24] and references therein). The same cannot be said of D -dimensional spacetimes, but recently there has been significant progress in this field. For instance, Refs. [25, 26] investigated gravitational radiation and the quadrupole formalism in higher-dimensional, asymptotically flat spacetimes. These studies showed that odd- and even-dimensional spacetimes behave differently, but there are simple energy formulas in the Fourier-domain that apply to both cases [26].

Linearized perturbations of higher-dimensional BHs are now well understood [27–30]. Historically, perturbative methods such as the close-limit approximation [32] (recently extended to higher dimensions [33, 34]) have provided guidance and insight in the numerical analysis of BH mergers in general relativity. The application of higher-dimensional BH perturbation theory to compute gravitational radiation in situations of physical interest was initiated in Ref. [31] (henceforth Paper I), where the authors studied the radiation produced by ultrarelativistic particles falling into even-dimensional, nonrotating, asymptotically flat BHs.

Numerical codes to evolve the Einstein equations in higher dimensions are presently capable of handling low-energy BH collisions in five dimensions [21]. However, the extension of these results to high-energy collisions in spacetimes of generic dimensionality presents a significant challenge. Motivated by these developments, here we extend the analysis of Paper I to study the energy and linear momentum radiated when particles of *arbitrary energy* fall into nonrotating, higher-dimensional BHs. Our results are in remarkable agreement with five-dimensional simulations of unequal-mass BH collisions in higher dimensions, that will be reported elsewhere [35]. They also provide useful (and sometimes surprising) insight into the

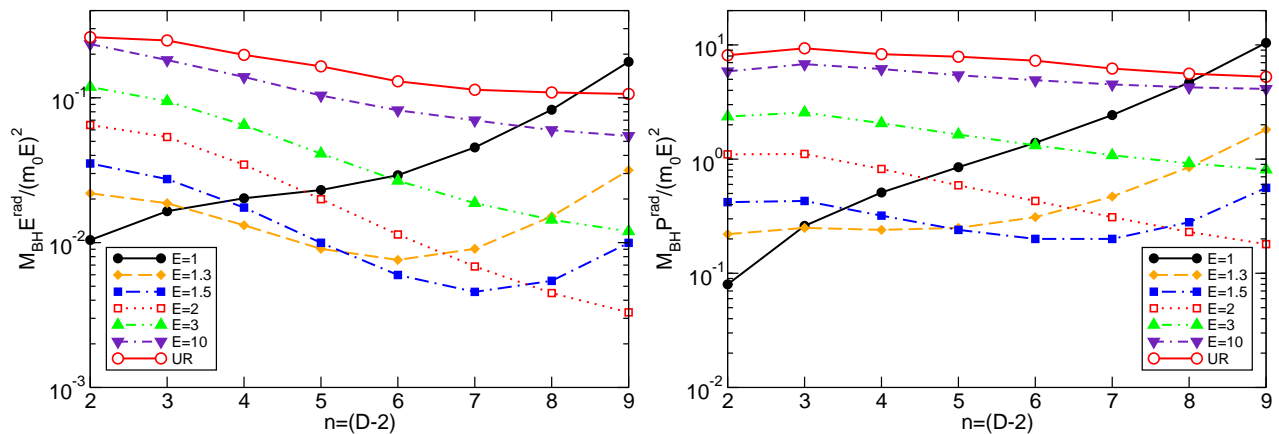


FIG. 1. Energy and momentum radiated plotted as functions of n for selected values of E .

energy- and dimensionality-dependence of gravitational radiation produced by head-on BH collisions.

The main findings of this paper are summarized in Figure 1, where we show the radiated energy (left) and linear momentum (right) as a function of $n = D - 2$ for selected values of the particle energy per unit mass E ($E = 1, 1.3, 1.5, 2, 3, 10$). Infalls from rest correspond to $E = 1$, and the ultrarelativistic case $E \rightarrow \infty$ is denoted by “UR” in the legend. As natural in perturbation theory, the energy and angular momentum radiated are inversely proportional to the BH mass M_{BH} and proportional to the square of the particle energy in the UR limit, so in the plot we normalize the radiation to $(m_0 E)^2 / M_{\text{BH}}$. Paper I found that the radiated energy *decreases* with n for ultrarelativistic infalls with $n \leq 8^1$. Figure 1 shows that the total energy radiated *increases* with n for particles falling from rest. Our results for $E = 1$ and $n = 3$ are in remarkably good agreement with numerical simulations of low-energy, unequal-mass BH collisions in $D = 5$ [35]. They should also provide a useful benchmark for future nonlinear evolutions of the Einstein equations in higher dimensions.

Even more interestingly, in some cases the left panel of Figure 1 shows the existence of a *local minimum* of the radiation as a function of n . This minimum is visible in the plot for the cases when the infall is not kinetic-energy dominated ($E = 1.3$ and $E = 1.5$), but we verified that it also occurs for $E = 2$ by extending our calculation to $n = 11$. *We conjecture that such a local minimum exists for any $E > 1$, and that the radiated energy may generically increase for sufficiently large n , eventually violating*

the point-particle approximation and the area theorem bound. Past work showed that point-particle results in four dimensions can be successfully extrapolated to the comparable-mass case (see e.g. [36]). Our results imply that for large n this will no longer be the case. In fact, we find that when $n = 11$ the radiation emitted in infalls from rest is *larger* than the particle mass, and therefore the point-particle approximation must break down. Incidentally, this breakdown fits in nicely with Hawking’s area bound [37]. Hawking’s area theorem, applied to infalls from rest in generic spacetime dimension n , predicts that the amount of radiation emitted in equal-mass BH collisions decreases with n . Our results do not violate the area theorem. Instead, they suggest a failure of the point-particle approach for a change in behavior of the total radiated energy (in the equal-mass case) for $n \gtrsim 11$. Unlike *Spinal Tap*’s Nigel Tufnel, in higher-dimensional gravitational radiation we can never “go to eleven”.

The plan of the paper is as follows. In Section II we briefly recall how to compute the radiation produced by particles falling radially (but with arbitrary energy) into a higher-dimensional BH. In Section III we present our results on the radiated energy and linear momentum (“kick velocity”), along with a preliminary comparison with numerical relativity results that will be presented in a companion paper. Section IV contains conclusions and possible directions for future research. In Appendix A we collect, for reference, some technical results.

II. FORMULATION OF THE PROBLEM

The spherically symmetric BH in $D = n + 2$ dimensions is described by the Schwarzschild-Tangherlini metric [38]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_n^2, \quad (1)$$

¹ An apparent exception to this rule is the case $n = 8$ in Table VI of Paper I. Unfortunately, the extrapolated energy for $n = 8$ ($D = 10$) was overestimated by $\sim 20\%$ in that paper. The reason is that we “only” computed multipoles up to $l = 20$ to estimate the total radiation, and as it turns out, this was not enough to get a reliable extrapolation of the total radiated energy. This error has been fixed here (see Table I below).

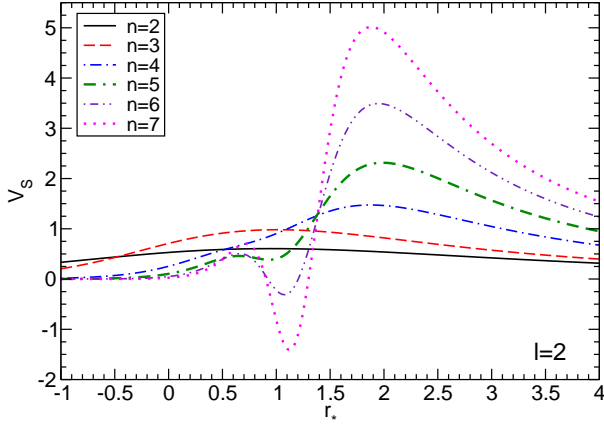


FIG. 2. The scalar potential of Eq. (6) as a function of the tortoise coordinate r_* for $l = 2$ and selected values of n . We use units $r_h = 1$.

where $d\Omega_n$ is the metric of the n -dimensional unit sphere S^n , and

$$f(r) = 1 - \frac{2M}{r^{n-1}}. \quad (2)$$

The BH mass is related to the parameter M by

$$M_{\text{BH}} = \frac{nM\mathcal{A}_n}{8\pi c^2 G_{n+2}}, \quad (3)$$

where $\mathcal{A}_n = 2\pi^{(n+1)/2}/\Gamma[(n+1)/2]$ is the area of S^n , G_{n+2} is the $(n+2)$ -dimensional Newton constant, and c is the speed of light. We will set $G_{n+2} = 1$ and $c = 1$ in the following. The tortoise coordinate r_* is defined by

$$\frac{dr_*}{dr} = \frac{1}{f(r)}. \quad (4)$$

An analytical expression for $r_*(r)$ valid for generic n is given in Paper I, Eqs. (5) and (6). Here and throughout the paper we use the notation of Ref. [27].

The computation of the gravitational wave emission of an ultrarelativistic particle plunging into a BH requires

the numerical integration of the inhomogeneous wave equation for scalar gravitational perturbations (“vector” and “tensor” gravitational perturbations, in the terminology of Kodama and Ishibashi, are not excited by a particle in radial infall). Setting $x \equiv 2M/r^{n-1}$, the equation for the scalar perturbations is

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V_S\right)\Phi_l^{(n)} = S_l^{(n)}. \quad (5)$$

where the scalar potential V_S is plotted in Figure 2 for selected values of n and $l = 2$. This potential is given by

$$V_S = \frac{f(r)Q(r)}{16r^2H(r)^2}, \quad (6)$$

where the function

$$H(r) = m + \frac{n(n+1)x}{2} \quad (7)$$

with $x = 2M/r^{n-1}$, $m = \kappa^2 - n$, $\kappa^2 = l(l+n-1)$, and

$$\begin{aligned} Q(r) = & n^4(n+1)^2x^3 + n(n+1) \times \\ & \times [4(2n^2 - 3n + 4)m + n(n-2)(n-4)(n+1)]x^2 \\ & - 12n[(n-4)m + n(n+1)(n-2)]mx \\ & + 16m^3 + 4n(n+2)m^2. \end{aligned} \quad (8)$$

To simplify the notation, below we will omit the superscript (n) from the wavefunction $\Phi_l^{(n)}$.

Equation (5) reduces to the inhomogeneous Zerilli equation [39] for $n = 2$. The source term $S_l^{(n)}$ in $(n+2)$ dimensions can be calculated from the stress-energy tensor of the infalling particle. Denote by E the particle energy per unit mass. Making use of the geodesic equations for massive particles in radial infall

$$\frac{dt}{d\tau} = \frac{E}{f(r)}, \quad \frac{dr}{d\tau} = -\sqrt{E^2 - f(r)}, \quad (9)$$

a straightforward generalization of the calculation presented in Paper I yields

$$S_l^{(n)} = \sqrt{32\pi}m_0S^{nl}e^{i\omega t(r)}\frac{f(r)}{r^{n/2}H}\left\{\frac{E}{i\omega r}\left(4 - \frac{n^2(n+1)[1-f(r)] - 2(n-2)m}{H}\right) + \frac{2}{\sqrt{E^2 - f(r)}}\right\}. \quad (10)$$

The normalized Gegenbauer polynomials S^{nl} are listed for the relevant values of n in Appendix A, along with simplified expressions of the source term in the ultrarelativistic case ($E \rightarrow \infty$).

We use a straightforward modification of the FORTRAN code described in Paper I to solve Eq. (5) via Green’s function techniques. We refer the reader to that paper for details. Just like in Paper I, for convenience, we set

the horizon radius $r_h = (2M)^{1/(n-1)} = 1$ in our numerical integrations. The energy spectrum can be expressed in terms of the wave amplitude at infinity Φ_l , given in Eq. (20) of Paper I, as

$$\frac{dE_l}{d\omega} = \frac{\omega^2}{16\pi} \frac{n-1}{n} \kappa^2 (\kappa^2 - n) |\Phi_l|^2. \quad (11)$$

Paper I did not provide a calculation of the radiated

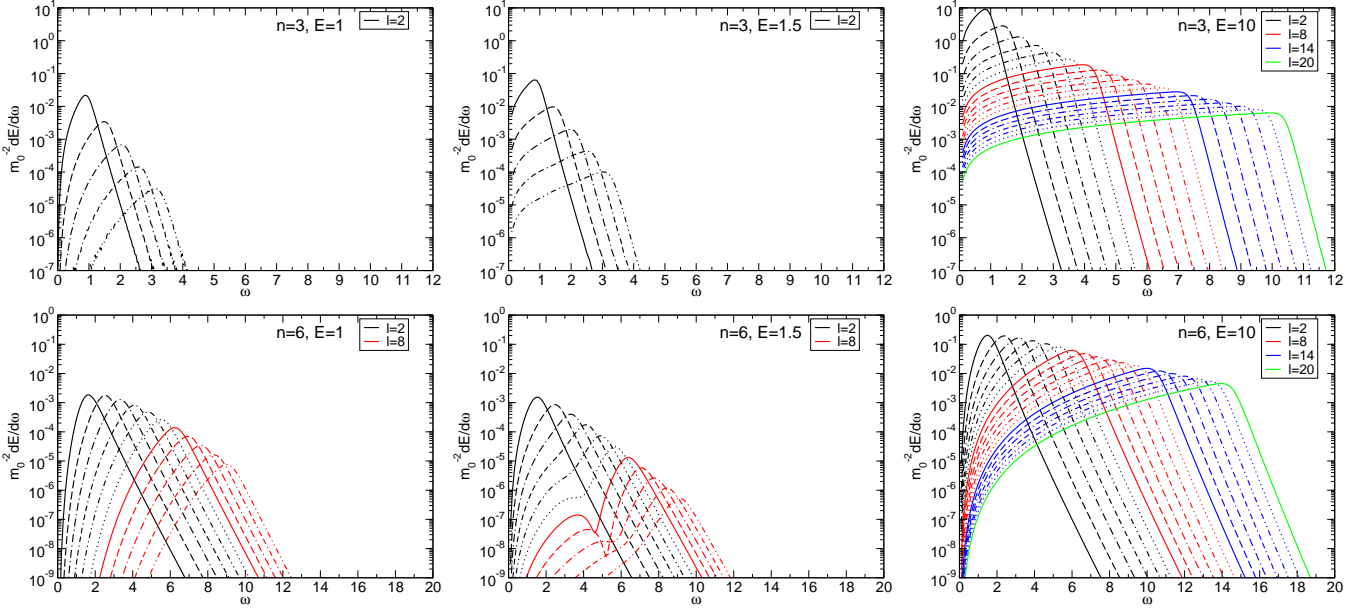


FIG. 3. Energy spectra for $n = 3$ and $n = 6$ (in units $r_h = 1$).

linear momentum P^i . The spectrum of the radiated momentum can be obtained from

$$\frac{dP^i}{d\omega} = \int_{S_\infty} d\Omega \frac{d^2 E}{d\omega d\Omega} n^i, \quad (12)$$

with n^i a unit radial vector on the sphere at infinity S_∞ . This results in an infinite series coupling different multipoles. Using only the first two terms in the series, we find for instance

$$\frac{dP^z}{d\omega} = 3\omega^2 \frac{\sqrt{5}(\Phi_3\Phi_2^* + \Phi_3^*\Phi_2) + 10(\Phi_3\Phi_4^* + \Phi_3^*\Phi_4)}{4\pi\sqrt{7}} \quad (13)$$

and

$$\frac{dP^z}{d\omega} = \omega^2 \frac{5(\Phi_3\Phi_2^* + \Phi_3^*\Phi_2) + 21(\Phi_3\Phi_4^* + \Phi_3^*\Phi_4)}{4\pi} \quad (14)$$

in $D = 4$ and $D = 5$, respectively. Here, Φ_l denotes the l -pole component of the Kodama-Ishibashi wavefunction and an asterisk denotes complex conjugation. We are assuming one-sided spectra. To get the total radiated linear momentum P^{rad} , in this work we do *not* truncate the series at the order shown in Eqs. (13) and (14). Instead we sum the required number of multipoles (typically $\sim 10 - 20$) to get the desired accuracy.

III. RESULTS

Our FORTRAN code passed several code checks. The spectra for $n = 2$ are in excellent agreement with those of Refs. [24, 40] for generic energies, and with those of

Ref. [41] in the ultrarelativistic limit; they have been reported several times in the literature, so we do not reproduce them here. Our even-dimensional ultrarelativistic spectra obviously reduce to those shown in Paper I. Results from the FORTRAN code were also verified by comparison with a MATHEMATICA notebook.

A. Energy

Figure 3 shows representative energy spectra for $n = 3$ and $n = 6$ at different values of the particle energy. In the ultrarelativistic limit, as pointed out analytically in Ref. [25] using Weinberg's "zero-frequency limit" approximation and confirmed numerically in Paper I, at low frequencies the spectra grow like ω^{n-2} , then they fall off exponentially beyond a cutoff frequency ω_c corresponding to the fundamental quasinormal mode frequency for the multipole in question (cf. Figure 1 in Paper I). This can be understood in terms of gravitational-wave scattering from the potential barrier surrounding the BH. The quantity ω^2 plays the role of the energy in the Schrödinger-like equation (5), so ω^2 is equal to the maximum of the scalar potential V_0 at first order in the WKB approximation. Therefore, only the radiation with energy smaller than the peak of the potential is backscattered to infinity; radiation with larger frequency is exponentially suppressed. This interpretation explains the salient features of Figure 3, and it is useful even in the context of comparable-mass, ultrarelativistic BH collisions [24].

A curious new feature of the energy spectra for $n \geq 5$ is the appearance of a double peak for large multipole number and intermediate particle energies. We have no quantitative explanation for these double peaks, but we

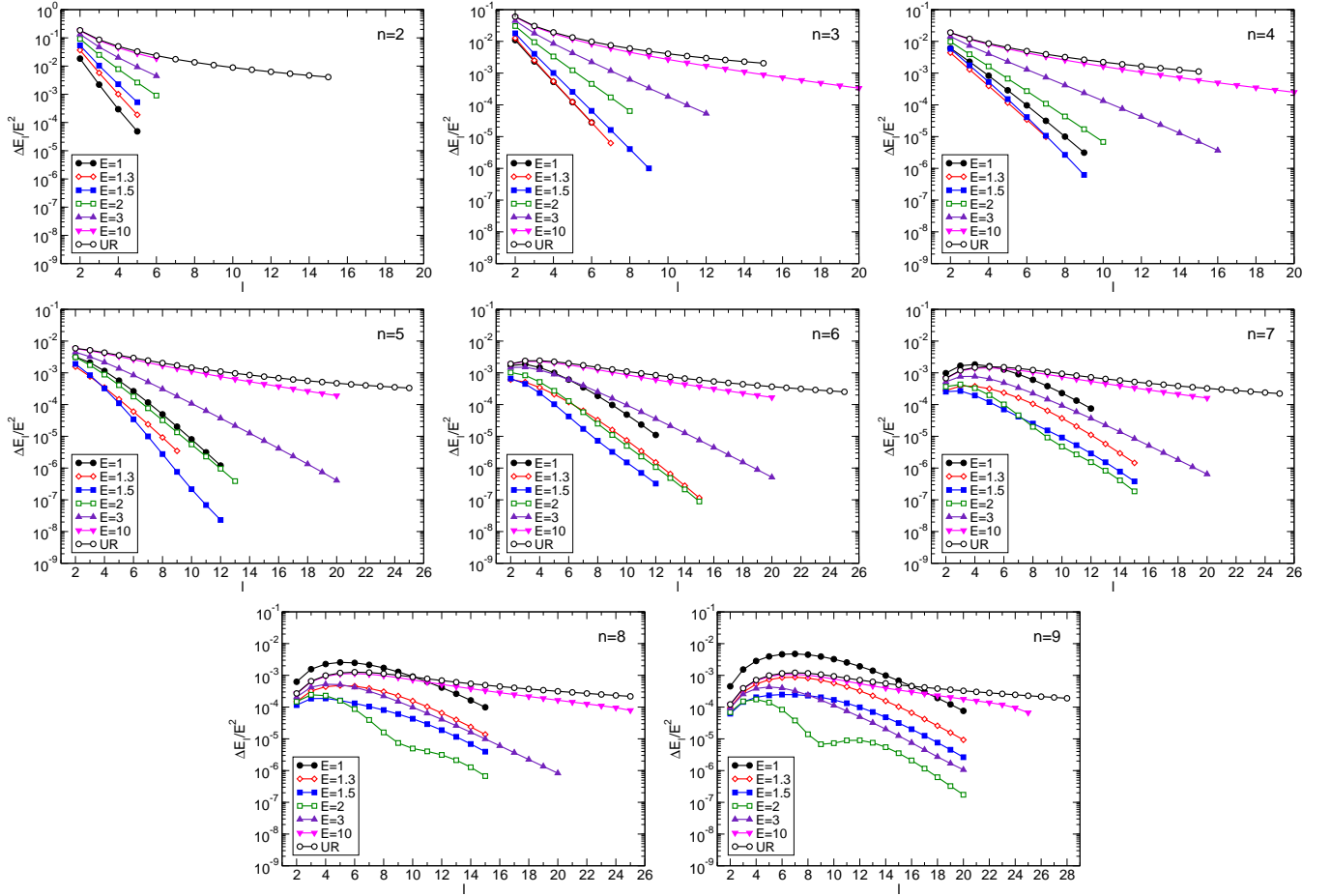


FIG. 4. Multipolar distribution for $n = 3, \dots, 9$.

suspect that they may be somehow related to the appearance of multiple peaks in the scalar potential for low values of l (cf. Figure 2).

For a given particle energy, higher multipoles contribute more as n grows. This is even more evident when we look at the ω -integrated multipolar components of the energy spectra of Figure 4. Starting from $n = 6$, in general the dominant multipole is no longer the quadrupole.

The total emitted energy is obtained by numerically integrating the spectra over ω and then by summing the individual multipolar components ΔE_l , which are shown in Figure 4. In principle, to compute the total energy we need to carry out a sum of all values of l up to $l \rightarrow \infty$. It is of course impossible to compute multipolar contributions ΔE_l for all values of l , so we computed a large enough number of multipoles for any given dimensionality n and particle energy E . In practice, for large l we fit the integrated ΔE_l with a power law of the form

$$\Delta E_l = a_{n+2} l^{-b_{n+2}}, \quad (15)$$

where the coefficients (a_{n+2}, b_{n+2}) are obtained by fitting (typically) the last five data points of each multipolar distribution in Figure 4. For each n and E , the

number of multipoles shown in the figure was chosen to minimize the dependence of these fits (and of the resulting extrapolation) on the specific values of l chosen for the fit. This extrapolation introduces larger uncertainties when E and/or n get large. Our final results are summarized in Table I and in the left panel of Figure 1.

For $E = 1$ (infall from rest) our results are well fitted by an expression of the form

$$10^2 \times \frac{M_{\text{BH}}}{m_0^2} E^{\text{rad}} = c_1 + c_2 \times c_3^D, \quad (16)$$

where $c_1 = 1.865$, $c_2 = 8.037 \times 10^{-4}$ and $c_3 = 2.457$. Now, based solely on the amount of emitted energy, one might expect the point-particle approximation to break down when $E^{\text{rad}} > m_0$. Based on the extrapolation of Eq. (16), this effectively constrains the mass ratio of the system to values $m_0/M_{\text{BH}} < 1$ when $D > 13$. For smaller D such a constraint does not apply. This may help to explain some results in the literature. For instance, consider the good agreement between numerical relativity simulations of equal-mass BH collisions and the point-particle extrapolations to equal-mass systems. In $D = 4$, early work [42] and more recent simulations (see e.g. [21]) found that the

TABLE I. Total energy radiated computed using the extrapolation of Eq. (15). For $E = 2$ we actually extended the calculation up to $n = 11$, and we found that a local minimum in the radiation occurs at $n = 10$: the corresponding entries in this table would be 0.326 ($n = 10$) and 0.575 ($n = 11$). For $E = 1$ and $n = 11$ the corresponding entry in this table would be 123, so the assumptions underlying our calculation are invalid (see text).

$10^2 \times (M_{\text{BH}} E^{\text{rad}})/(m_0^2 E^2)$							
n	$E = 1$	$E = 1.3$	$E = 1.5$	$E = 2$	$E = 3$	$E = 10$	UR
2	1.04	2.19	3.52	6.49	11.9	23.5	26.2
3	1.65	1.87	2.75	5.36	9.47	18.2	24.9
4	2.02	1.32	1.75	3.46	6.48	13.9	19.8
5	2.31	0.905	1.00	1.99	4.11	10.4	16.5
6	2.92	0.760	0.598	1.14	2.67	8.20	13.0
7	4.54	0.906	0.457	0.684	1.88	6.99	11.4
8	8.27	1.52	0.545	0.449	1.44	5.98	10.9
9	17.7	3.16	1.00	0.330	1.20	5.45	10.6

energy radiated in full nonlinear simulations of head-on collisions with finite mass ratio is well reproduced by simply taking the *linearized* result for a particle falling into a BH and replacing $m_0 \rightarrow \mu$, where μ is the reduced mass of the system. This surprising agreement is at least consistent with the observation that radiation is weak, and therefore nonlinear effects are small.

However, as D grows the amount of radiation emission also grows (a similar effect was observed recently in other settings [43, 44]). The extrapolation of perturbative results to finite mass ratio must eventually break down, for the following reason. For two equal-mass BHs, Hawking's area theorem implies that the area of the final BH must be equal to or larger than that of the initial BHs. This implies the following bound on radiation emission [21]

$$\frac{E^{\text{area}}}{2M_{\text{BH}}} \leq 1 - 2^{-\frac{1}{D-2}}. \quad (17)$$

Now, this bound decreases with dimension and will eventually be a strong restriction to the amount of radiation, violating the extrapolation of Eq. (16) to the equal-mass case. In fact, Hawking's area theorem is more restrictive than Eq. (16) for $D > 13$ – precisely when we know that the point-particle approximation breaks down due to the large amount of gravitational radiation.

Summarizing, our results are self-consistent, they are consistent with the area theorem bound, and moreover they predict a qualitative change in the equal-mass BH collision as D increases: for $D \leq 13$ the amount of energy lost as gravitational waves increases with D , but our calculation breaks down at $D = 13$, and presumably for higher dimensions the radiated energy may start to decrease to conform with the area bound.

Ongoing simulations of head-on BH collisions from rest ($E = 1$) in $D = 5$ consider unequal-mass BHs with mass

ratios $q = m_1/m_2$ in the range between 1 and $1/4$ [35]. Extrapolation of the numerical results to the extreme mass ratio limit yields [35]

$$E^{\text{rad}} = 0.0164 \frac{m_0^2}{M_{\text{BH}}}. \quad (18)$$

This prediction agrees within better than 1% with the $n = 3$, $E = 1$ prediction listed in Table I. This excellent agreement provides a strong sanity check of the complex numerical relativity simulations, and a useful example of the significance of point-particle calculations such as those presented here. A thorough analysis of the nonlinear simulations (including more extensive comparisons with the point-particle limit) is in preparation [35].

B. Linear momentum

TABLE II. Momentum radiated.

$10^2 \times (M_{\text{BH}} P^{\text{rad}})/(m_0^2 E^2)$							
n	$E = 1.0$	$E = 1.3$	$E = 1.5$	$E = 2$	$E = 3$	$E = 10$	UR
2	0.082	0.22	0.42	1.1	2.4	5.9	8.1
3	0.26	0.25	0.43	1.1	2.6	6.8	9.3
4	0.51	0.24	0.32	0.82	2.1	6.2	8.3
5	0.85	0.25	0.24	0.59	1.6	5.4	7.9
6	1.4	0.31	0.20	0.43	1.3	4.9	7.3
7	2.4	0.47	0.20	0.31	1.1	4.5	6.2
8	4.7	0.85	0.28	0.23	0.92	4.2	5.6
9	10	1.8	0.56	0.18	0.81	4.1	5.3

In Table II and in the right panel of Figure 1 we summarize the results for the linear momentum emitted in gravitational waves. The pattern for momentum emission closely mimics that of energy emission. If perturbative results can be extrapolated to finite mass ratios (which is the case for lower spacetime dimensions, see Ref. [35]) one expects the following mass ratio dependence [45]:

$$P = A_v(m_0 E) \frac{q(1-q)}{(1+q)^5}, \quad (19)$$

where $q \equiv m_0 E/M_{\text{BH}}$. The quantity A_v can be read off from Table II in the small- q limit. From the momentum, one can get the recoil velocity

$$\frac{v_{\text{kick}}}{c} = A_v \frac{q^2(1-q)}{(1+q)^5}. \quad (20)$$

This equation predicts a maximum kick velocity $v_{\text{kick}}^{\text{max}}/c = 0.0179 A_v$ for $q = (3 + \sqrt{5})/2 \sim 0.382$. Numerical simulations of BH collisions from rest in $D = 5$ indicate that, in the point-particle limit [35],

$$\frac{v_{\text{kick}}}{c} = 0.24 \frac{m_0^2}{M_{\text{BH}}}, \quad (21)$$

again in very good agreement with Table II. This is a nontrivial test of the simulations, because the emission of linear momentum involves interference between different multipoles.

IV. CONCLUSIONS AND OUTLOOK

Our results for the energy spectrum, total energy and momentum radiated during the head-on infall of a point particle into a higher-dimensional BH show an interesting and complex structure. The results indicate a beautiful concordance with the area theorem and they suggest that the extrapolation of perturbation theory to equal-mass collisions will yield wrong results for dimensions $D \gtrsim 13$. This suggests that there should be a mechanism suppressing the total amount of radiation in large spacetime dimensions. Full nonlinear evolutions of the Einstein equations will probably be needed to clarify the exact nature of this mechanism.

A natural and interesting generalization of our results would be to study the large- D limit with either numerical or analytical techniques. Other obvious generalizations include the study of infalls with finite impact parameters and of rotating (Myers-Perry) black holes.

The present results should be relevant to the nascent field of numerical relativity in higher-dimensional spacetimes. They can be used as a guide and benchmark for future nonlinear simulations. Indeed, we will show in forthcoming work how full numerical simulations of Einstein's equations are remarkably consistent with the results reported here [35].

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Appendix A: Normalization coefficients and ultrarelativistic limit of the source term

For the reader's convenience, here we list the normalized Gegenbauer polynomials \mathcal{S}^{nl} appearing in Eq. (10)

for the relevant values of n and $\theta = 0$:

$$\begin{aligned} \mathcal{S}^{2l} &= \frac{1}{2} \sqrt{\frac{2l+1}{\pi}}, \quad \mathcal{S}^{3l} = \frac{l+1}{\pi\sqrt{2}}, \\ \mathcal{S}^{4l} &= \frac{1}{4\pi} \sqrt{(2l+3)\lambda_2}, \quad \mathcal{S}^{5l} = \frac{1}{2\pi} \sqrt{\frac{(l+2)\lambda_3}{3\pi}}, \\ \mathcal{S}^{6l} &= \frac{1}{8\pi} \sqrt{\frac{(2l+5)\lambda_4}{2\pi}}, \quad \mathcal{S}^{7l} = \frac{1}{4\pi^2} \sqrt{\frac{(2l+6)\lambda_5}{15}}, \\ \mathcal{S}^{8l} &= \frac{1}{16\pi^2} \sqrt{\frac{(2l+7)\lambda_6}{6}}, \quad \mathcal{S}^{9l} = \frac{1}{4\pi^2} \sqrt{\frac{(l+4)\lambda_7}{105\pi}}, \\ \mathcal{S}^{10l} &= \frac{1}{64\pi^2} \sqrt{\frac{(2l+9)\lambda_8}{6\pi}}, \quad \mathcal{S}^{11l} = \frac{1}{24\pi^3} \sqrt{\frac{(2l+10)\lambda_9}{105}}, \end{aligned} \quad (\text{A1})$$

where $\lambda_k \equiv (l+k)(l+k-1)\dots(l+1)$.

In the ultrarelativistic limit $E \rightarrow \infty$, the definition of the tortoise coordinate implies that $t(r) = -r_*(r)$. In units $r_h = (2M)^{1/(n-1)} = 1$, the source term (10) reduces to:

$$\begin{aligned} S_l^{(2)} &= e^{-i\omega r_*} \frac{8\sqrt{4l+2}}{i\omega r} \frac{(r-1)\nu_2}{[\nu_2 r + 3]^2}, \\ S_l^{(3)} &= e^{-i\omega r_*} \frac{24(l+1)}{i\omega r^2 \sqrt{\pi r}} \frac{[(r^4 - r^2)\nu_3 + 2(1 - r^2)]}{[\nu_3 r^2 + 6]^2}, \\ S_l^{(4)} &= e^{-i\omega r_*} \frac{16\sqrt{\lambda_2(2l+3)}}{i\omega r^3 \sqrt{2\pi}} \frac{[(r^6 - r^3)\nu_4 + 5(1 - r^3)]}{[\nu_4 r^3 + 10]^2}, \\ S_l^{(5)} &= e^{-i\omega r_*} \frac{20\sqrt{\lambda_3(2l+4)}}{i\omega \pi r^3 \sqrt{3r}} \frac{[(r^8 - r^4)\nu_5 + 9(1 - r^4)]}{[\nu_5 r^4 + 15]^2}, \\ S_l^{(6)} &= e^{-i\omega r_*} \frac{6\sqrt{\lambda_4(2l+5)}}{i\omega \pi r^4} \frac{[(r^{10} - r^5)\nu_6 + 14(1 - r^5)]}{[\nu_6 r^5 + 21]^2}, \\ S_l^{(7)} &= e^{-i\omega r_*} \frac{28\sqrt{\lambda_5(2l+6)}}{i\omega \pi r^4 \sqrt{30\pi r}} \frac{[(r^{12} - r^6)\nu_7 + 20(1 - r^6)]}{[\nu_7 r^6 + 28]^2}, \\ S_l^{(8)} &= e^{-i\omega r_*} \frac{4\sqrt{\lambda_6(2l+7)}}{i\omega \pi r^5 \sqrt{3\pi}} \frac{[(r^{14} - r^7)\nu_8 + 27(1 - r^7)]}{[\nu_8 r^7 + 36]^2}, \\ S_l^{(9)} &= e^{-i\omega r_*} \frac{18\sqrt{\lambda_7(2l+8)}}{i\omega \pi^2 r^5 \sqrt{105r}} \frac{[(r^{16} - r^8)\nu_9 + 35(1 - r^8)]}{[\nu_9 r^8 + 45]^2}, \\ S_l^{(10)} &= e^{-i\omega r_*} \frac{5\sqrt{\lambda_8(2l+9)}}{i\omega 4\pi^2 r^6 \sqrt{3}} \frac{[(r^{18} - r^9)\nu_{10} + 44(1 - r^9)]}{[\nu_{10} r^9 + 55]^2}, \\ S_l^{(11)} &= e^{-i\omega r_*} \frac{22\sqrt{\lambda_9(2l+10)}}{i\omega 3\pi^2 r^6 \sqrt{210\pi r}} \\ &\quad \times \frac{[(r^{20} - r^{10})\nu_{11} + 54(1 - r^{10})]}{[\nu_{11} r^{10} + 66]^2}, \end{aligned} \quad (\text{A2})$$

where $\nu_k = (l+k)(l-1)$. These expressions are consistent with those listed (for even dimensions) in Paper I.

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